

Optimal Unemployment Insurance with Consumption Commitments

-- Can Current UI Policy Be Justified?

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[Abstract] Unemployment insurance programs are important ingredients of social welfare policies in developed countries. Over the past two decades, there has been a branch of literature that deals with the optimal design of insurance plans. However, no theory so far has been able to justify the current UI program in the U.S. Classical models claim that unemployment benefits should decrease gradually in order to induce the appropriate incentive to be reemployed, while the current program is a flat replacement ratio of approximate 66% for 6 months that then drops down to zero afterwards. This paper considers an environment with consumption commitments in which people cannot freely substitute among different goods within a single period. In this paper, the optimal unemployment contract is designed in a repeated principal-agent problem with unobservable job search effort. With consumption commitments, the optimal plan involves a relatively flat decreasing sequence of insurance payments over some duration, which is then followed by a large drop to a very low level of transfer. The results fit current policy well, and therefore give an explanation to justify the current policy. Additionally, the model predicts that if we change from the current unemployment insurance program to the optimal contract, the government will only save 1.7% in unemployment insurance payments, which shows that current policy is not as flawed as researchers have traditionally believed. In fact, to achieve efficiency, unemployment transfers should include a jump, similar to what we observe in practice. The difference between the optimal contracts I obtain in theory and the current policy in the real world is small and can be explained by administration cost.

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I. Introduction

Unemployment insurance (UI) program is an important ingredient of social welfare policies in developed countries and they vary across countries and over time in terms of their size and duration. In the U.S., unemployment benefits take the form of approximate 66% replacement ratio for 6 months which then drop down to zero afterwards.

However, current UI policy has been widely criticized because of its perverse effects on the incentives for reemployment. Over the past two decades, there has been a branch of literature that deals with the optimal design of insurance plans. There is a big discrepancy between the optimal plans proposed theoretically and what we observe in practice. For example, classical models that focus on unobservable job search effort argue that unemployment benefits should decrease gradually in order to induce the appropriate incentive to be reemployed. So far no theory has been able to justify current UI policy, where we have a flat insurance payment for a fixed duration followed by a large drop in benefits.

The purpose of this paper is to give an explanation for the existence of discontinuity in the sequence of insurance payments. We want to show that the current unemployment policy isn't as bad as researchers have traditionally expected; actually the efficient unemployment transfers should include a large drop as what we observe in practice.

We model the incentive problem caused by unemployment insurance in a repeated principal-agent problem where the moral hazard comes from unobservable job search effort. The innovation of this paper is that we design the optimal unemployment insurance contracts within the environment in which people cannot substitute among different goods freely within a period. We propose this setting because in the real world, many goods, such as housing and vehicles, involve "commitments"-- adjustment costs have to be paid to change the consumption of these goods.

There is a contract between the principal (government) and the agent (unemployed worker) that specifies a sequence of payments from the principal to the agent conditional

on the agent's previous unemployment history and the commitment goods he previously had. The optimal contract minimizes the expected discounted costs, which include both transfers and the adjustment costs if commitment goods are changed, subject to the constraint that the unemployed are ensured a prespecified welfare level. We know that optimal UI is to strike a balance between the benefit of consumption smoothing and the cost of the insurance. The point is when we incorporate consumption commitments into the model; the adjustment among consumption goods is not completely flexible any longer. Therefore the demand for consumption smoothing would change. In this case, we need to reconsider the optimal unemployment insurance plan.

The optimal plan I obtained has a relatively flat decreasing insurance payment from the government to the unemployed during periods of unemployment for some duration then followed by a significant drop to a very low level of transfers. The results fit current UI policy much better than those of classical models. The logic behind my results is as follows: first, in order to provide inter-temporal incentives for job search, the contract must punish workers for continuing to be unemployed by reducing their claims for future consumption. This is the reason for decreasing transfers during periods of unemployment; second, the decreasing rate is much slower than those of classical models. This is because consumption commitments change preference risk aversion. To put it concretely, let's use food and housing to represent adjustable goods and commitment goods respectively. Suppose the agent suffers an unemployment shock. Before he decides to pay the adjustment cost and sell his house, he has to cut back sharply on food expenditure in order to pay his mortgage and maintain the housing commitment that he previously made. This will amplify his risk aversion, because the sharp concentrated reduction in food expenditure creates a larger increase in marginal utility than would occur without commitments, where housing expenditure could also be cut. Therefore, as long as commitment goods are maintained, the unemployed are worse off than we have expected. Their demands for consumption smoothing are larger, which requires a higher level of unemployment benefits; and last, the discontinuity pattern is due to the adjustment of commitment goods. Once the commitment goods are downgraded, the agent has more flexibility in adjusting his consumption bundle. The government no longer needs to

maintain such a high level of insurance payment; therefore a drop in optimal unemployment insurance is expected.

The paper proceeds as follows. Section II gives a brief literature review and some facts about consumption commitments. In section III I describe the model and state some theoretical results. Numerical results and sensitivity analysis are provided in section IV. Section V consists of the conclusion and extensions.

II. Literature Review and Facts

Literature Review

The literature on optimal UI is relatively new. Much of research on optimal unemployment insurance design has been devoted to the study of the effect of UI benefits on the duration of unemployment spells, emphasizing the moral hazard created by insufficient monitoring of job search effort. The analysis of Shavell and Weiss (1979) focuses on the moral hazard associated with the inability of the UI provider to monitor the job search effort and reservation wage of the unemployed. The main insight of their paper is that the UI benefits should decline monotonically with the duration of unemployment spells. Building upon this work, Hopenhayn and Nicolini (1997) discuss the design of an optimal UI program in which the insurer's power to reward or punish includes the ability to tax or supplement the agent's income after he is reemployed. The key feature of their model is that the likelihood of finding a new job in any given period depends on the search effort of the unemployed, which is private information. They show that to motivate the unemployed to exert appropriate job search effort, the expected utility associated with remaining unemployed must decline over time. These conclusions seem compelling, however they are far from the UI program we have in practice. This paper is based on Hopenhayn and Nicolini (1997). I incorporate commitment goods into the optimal UI contract design, in which the adjustment of commitment goods is costly, the result of which is a discontinuity in unemployment benefits.

Consumption Commitments

In most existing models, we assume that individuals consume only one good, that is, consumption can be regarded as a composite good. Single-good utility function is under

the assumption that people can substitute freely among different goods at all times. In practice, however, substituting among goods within a period is costly.

Many households have “consumption commitments” that are costly to adjust when shocks such as job loss or illness occur. Gruber (1998) finds that less than 5% of the unemployed move out of their homes during unemployment and most of them still have mortgage commitments. Housing is a leading example of a commitment good. Changing consumption of housing typically entails moving expenses and large transaction costs. Broker fees are around 5% of a home’s listing price, and early termination of a lease often entails at least a one-month penalty in rent. In addition, leaving a neighborhood and changing children’s schools could have direct welfare costs. More generally, any durable good involves some commitment, as market resale values are significantly lower than actual values. The difference between the price of a new car and a one-year-old car suggests that the loss from reselling a durable good is at least 10% of its value, perhaps due to phenomenon of lemons. Additionally, a number of services require explicit contracts and penalties for early termination, such as: health insurance, health clubs, cellular phones, and cable television.

Goods involving commitments comprise a significant portion of most households’ budgets. In empirical studies, Warran and Tyagi (2003) show that nearly 70 percent of the average household’s budget is to some extent non-discretionary in the short run. Raj Chetty (2003) argues that the typical household in the US allocates approximately half of its net-of-tax income to these costly-to-adjust goods.

III. The Model

The Environment

In this section we characterize the optimal unemployment insurance contract between the government and the unemployed worker with consumption commitments.

The preferences of the agent are:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t, y_t) - a_t] \quad (1)^{1,2}$$

¹ We disregard the disutility from working when the agent is employed.

where $x_t \in R_+$ and $y_t \in R_+$ are adjustable and commitment good respectively at time t . $a_t \in A$ is search effort. $\beta < 1$ is a discount factor and E_0 is the expectation operator at $t = 0$. A is a closed interval containing zero. The utility function is increasing and strictly concave in both consumption goods: $u_1 > 0, u_2 > 0, u_{11} < 0, u_{22} < 0$.

ASSUMPTION1 $\lim_{x \rightarrow \infty} u_1(x, y) = \lim_{y \rightarrow \infty} u_2(x, y) = 0$ and $\lim_{x \rightarrow 0} u(x, y) = \inf_{x', y'} u(x', y')$ for all x .

ASSUMPTION2 The marginal utility of adjustable good is non-decreasing in consumption of commitment good: $u_{1,2}(x, y) \geq 0$.

Search effort a_t enters negatively in the utility function and it is private information of the unemployed worker. The likelihood of finding a new job p_t depends on a_t

$$p_t = p(a_t) \quad (2)$$

where $p(\cdot)$ is strictly increasing, concave, twice differentiable and satisfies standard Inada conditions to guarantee an interior solution.

In this model, we assume the agent has no other source of income except transfers from the government when they are unemployed. We also assume the agent has no savings and no access to credit. Under these assumptions, the government can directly control the agent's consumption stream. Here we are not going to consider the case where there is hidden wealth or trade such as borrowing and lending that cannot be observed by the government.³ We assume all jobs are identical, offering a permanent and constant wage. This assumption is just to exclude another kind of asymmetric information where the job offers are heterogeneous and cannot be observed. Lastly, we assume once commitment goods are downgraded, they can be reduced freely. This assumption is for simplicity.⁴ There are some reasons to justify it: suppose the worker owns a house when he becomes unemployed. When he downgrades from self-owned housing to apartment renting, the

² The utility form here is fairly standard: concavity, additive time separability, and additive separability between consumption and search effort.

³ Following Hopenhayn and Nicolini (1997) arguments: these assumptions are quite typical in the repeated agency literature and provide an upper bound on what can be achieved through an optimal contract.

⁴ I am going to adjust this assumption in future work.

adjustment cost cannot be ignored. But after that, as an apartment renter, moving from a big apartment to a small one induces a much smaller adjustment cost that could theoretically be neglected. Another example is selling a car and changing to public transportation. Once the commitment good (the car) is sold, the agent has almost complete flexibility to adjust the consumption bundle.

The Contract

Sequential Problem

At time $t = 0$ the government offers a contract to the unemployed worker. The contract is an allocation $\sigma = \{\tau_t(v_0, y_{-1}, h^t), x_t(v_0, y_{-1}, h^t), y_t(v_0, y_{-1}, h^t), a_t(v_0, y_{-1}, h^t)\}_{t=0}^{\infty}$, where v_0 is the initial welfare value for the agent at time $t = 0$. y_{-1} is the given level of commitment good the agent previously consumed. $h^t = (h_0, h_1, \dots, h_t)$ is the history up to period t . $h_t \in H = \{0, 1\}$, where 0 represents the state of unemployment and 1 stands for employed status. $\tau_t(v_0, y_{-1}, h^t)$ is the transfer from the government to individual at time t , which can be allocated on the consumption of adjustable good $x_t(v_0, y_{-1}, h^t)$ and commitment good $y_t(v_0, y_{-1}, h^t)$. $a_t(v_0, y_{-1}, h^t)$ is the recommended search effort by the government.⁵ Associated with each contract, let $C_0(\sigma)$ denote the expected discounted value of total cost, which includes both net transfers and adjustment cost if commitment good is changed. Let $V_0(\sigma) = E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t(\sigma), y_t(\sigma)) - a_t(\sigma)]$ represent discounted utility value for the agent corresponding to the contract allocation σ .

The optimal contract is to choose the efficient allocation σ to minimize the cost of unemployment insurance subject to the constraint that the unemployed are ensured the initial welfare value.⁶ The letter K is adjustment cost.

$$C_0(v_0, y_{-1}) = \underset{\{\tau_t, x_t, y_t, a_t\}_{t=0}^{\infty}}{\text{Min}} E_0 \sum_{t=0}^{\infty} \beta^t [\tau_t(v_0, y_{-1}, h^t) + 1(y_t(v_0, y_{-1}, h^t) \neq y_{-1})K] \quad (3)$$

s.t.

⁵ Relevant only if the worker is unemployed.

⁶ Assume the government discounted the future the same as the agent

Promise keeping constraint $\forall v_0$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(x_t(v_0, y_{-1}, h^t), y_t(v_0, y_{-1}, h^t)) - a_t(v_0, y_{-1}, h^t) \right] \geq v_0 \quad (4)$$

Incentive constraint $\forall h^t$

$$\left\{ a_v(v_0, y_{-1}, h^v) \right\}_{v=t}^{\infty} \in \arg \max_{\{a_v\}_{v=t}^{\infty}} E_t \sum_{v=t}^{\infty} \beta^v \left[u(x_v(v_0, y_{-1}, h^v), y_v(v_0, y_{-1}, h^v)) - a_v \right] \quad (5)$$

And

$$\tau_t(v_0, y_{-1}, h^t) = x_t(v_0, y_{-1}, h^t) + y_t(v_0, y_{-1}, h^t) \quad (6)^7$$

In this problem, we assume the government has the ability to commit to the transfer policy.

Recursive Problem

We write the optimal contract problem in a recursive form. The expected discounted utility to the agent is equal to v_0 . Each period, the decision of whether to change the commitment goods is made. If the commitment good is adjusted, government transfers τ to the unemployed which could be used on adjustment good x and commitment good y . The recommended search effort a during this period is specified in the unemployment contract. The contract also specifies the next period promised value v' to the agent. If the commitment good is maintained, contract includes transfer $\hat{\tau}$, search effort \hat{a} and promised value \hat{v}' for the next period. Because the commitment good is maintained, a fixed amount of transfer $\hat{\tau}$ goes to the commitment good, the level of which given as y_{-1} . The rest of transfer $\hat{\tau}$ is spent on adjustable good \hat{x} . The letter K is the adjustment cost. The optimal contract can be described as the minimizing problem followed:

$$C(v, y_{-1}) = \text{Min} \left\{ \begin{array}{l} \text{Min}_{\{\tau, x, y, a, v'\}} \left\{ \tau + 1(y \neq y_{-1})K + \beta(1 - p(a))W(v') \right\} \\ \text{Min}_{\{\hat{\tau}, \hat{x}, \hat{a}, \hat{v}'\}} \left\{ \hat{\tau} + \beta(1 - p(\hat{a}))C(\hat{v}', y_{-1}) \right\} \end{array} \right\} \quad (7)$$

s.t.

Promise keeping constraints $\forall v$

$$u(x, y) - a + \beta(p(a)v^e + (1 - p(a))v') \geq v \quad (8)$$

⁷ Assume price of commitment good is 1.

$$u(\hat{x}, y_{-1}) - \hat{a} + \beta[p(\hat{a})v^e + (1 - p(\hat{a}))\hat{v}'] \geq v \quad (9)$$

Incentive constraints

$$a \in \arg \max \{u(x, y) - a + \beta[p(a)v^e + (1 - p(a))v']\} \quad (10)$$

$$\hat{a} \in \arg \max \{u(\hat{x}, y_{-1}) - \hat{a} + \beta[p(\hat{a})v^e + (1 - p(\hat{a}))\hat{v}']\} \quad (11)$$

and

$$x + y = \tau \quad (12)$$

$$\hat{x} + y_{-1} = \hat{\tau} \quad (13)$$

where $C(v, y_{-1})$ is total cost given that the promised value is v and the commitment good is y_{-1} . $p(a)$ is the probability of finding a job during the next period. $W(v')$ is the discounted cost in the next period when the commitment good is adjusted during this period:⁸

$$W(v') = \underset{x', y', \tau', a', v'}{\text{Min}} \{ \tau' + \beta(1 - p(a'))W(v'') \} \quad (14)$$

s.t.

$$u(x', y') - a' + \beta(p(a')v^e + (1 - p(a'))v'') \geq v' \quad (15)$$

$$a' \in \arg \max_{a'} \{u(x', y') - a' + \beta(p(a')v^e + (1 - p(a'))v'')\} \quad (16)$$

$$x' + y' = \tau' \quad (17)$$

v^e is the discounted utility value for a worker who is employed:^{9,10}

$$v^e = \underset{x, y}{\text{Max}} \frac{u(x, y)}{1 - \beta} \quad (18)$$

$$\text{s.t. } x + y = w, u_1(x, y)p = u_2(x, y) \quad (19)$$

Since $p(a)$ is strictly concave, the incentive constraints (10) and (11) can be written as:¹¹

$$\beta p'(a)(v^e - v') = 1 \quad (20)$$

$$\beta p'(\hat{a})(v^e - \hat{v}') = 1 \quad (21)$$

⁸ Notice commitment good doesn't enter the cost function because the assumption that once the commitment good is downgraded, the agent has the flexibility to adjust consumption bundle afterwards.

⁹ v^e doesn't depend on unemployment history.

¹⁰ For simplicity, we assume there is no adjustment cost for an upgrading in commitment good. We can relax this assumption and assume an adjustment for upgrading, but this will only change the quantitative value of v^e , which will not change the properties of the optimal contract.

¹¹ As long as $v^e > v'$, a is greater than zero.

Characterization of the Optimal Contract

Since the cost function is the minimum of two minimizing problems, it is not smooth at the point when adjustment is made. But if we disregard this kink and look at other parts of the cost function, we can still use first-order conditions to get some properties of the optimal contract. Before the commitment good is adjusted, the first-order conditions are:

$$C'(\hat{v}', y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \frac{\beta \eta p'(\hat{a})}{(1 - p(\hat{a}))} \quad (22)$$

$$p'(\hat{a})C(\hat{v}', y_{-1}) + \hat{\eta} p''(\hat{a})(v^e - \hat{v}') = 0 \quad (23)$$

The envelope condition is:

$$C'(v, y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} = C'(\hat{v}', y_{-1}) + \frac{\beta \hat{\eta} p'(\hat{a})}{(1 - p(\hat{a}))} \quad (24)$$

where $\hat{\eta}$ is the multiplier on the incentive-compatibility constraint. We also derive the first-order conditions for the case in which the commitment good is adjusted. Since they are similar to equation (22)-(24), we put them in appendix [1].

PROPOSITION 1: The promised utility value in the next period is less than the promised value in this period, i.e., $\hat{v}' < v$ or $v' < v$, if cost function is convex.

Proof. First we consider the case in which the commitment good has not been adjusted. Since we have $p'(\cdot) > 0$, $p''(\cdot) < 0$ and $v^e > v'$, from (23) we get $\hat{\eta} > 0$. From (24) we obtain $C'(v, y_{-1}) > C'(\hat{v}', y_{-1})$. If the cost function C is convex, we get $v > \hat{v}'$. After the commitment good is adjusted, we can follow the first-order conditions in the appendix and the proof is similar. At the kink when adjustment is made, since we choose the minimum of two minimizing results, if $\hat{v}' < v'$, we have already proved that $v > \hat{v}'$, so the conclusion is valid; if $\hat{v}' > v'$, we have $v > \hat{v}' > v'$. Q.E.D.

The intuition of proposition 1 is that in order to provide proper incentive for seeking a job, the contract must punish workers for continued unemployment by reducing their claims for future consumption.

PROPOSITION 2: The replacement ratio $(\frac{\tau}{w})$ or $(\frac{\hat{\tau}}{w})$ is an increasing function of current state variable v .

Proof. Before the commitment is changed, from envelope condition (24), we get

$$C'(v, y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})}. \text{ When } v \downarrow, \text{ since } C \text{ is convex, we have } C'(v, y_{-1}) \downarrow, \text{ therefore}$$

$u_1(\hat{\tau} - y_{-1}, y_{-1}) \uparrow, \Rightarrow \hat{\tau} \downarrow$. Because we assume a permanent and constant wage when the worker is employed, we get that the replacement ratio decreases. After the adjustment, the proof is similar. Q.E.D.

LEMMA 1: The utility level in this period and the promised value in the next period move in opposite directions, given hazard rate function $p(a) = 1 - e^{-ra}$. If $\hat{v}'_1 > \hat{v}'_2$ then $u(\hat{x}_1, y_{-1}) < u(\hat{x}_2, y_{-1})$; if $v'_1 > v'_2$ then $u(x_1, y_1) < u(x_2, y_2)$.

Proof. See appendix [2].

PROPOSITION 3: Promised value next period (\hat{v}' or v') is an increasing function of current state v .

Proof. See appendix [3].

Proposition 2 and 3 are intuitive: the less utility the agent is promised today, the less the agent will get from the government for today and his claims for future consumption will be less.

COROLLARY 1: Transfer from the government to the agent is decreasing during the unemployment period.

Proof. Follows immediately by repeatedly applying proposition 1 and 2.

Corollary 1 states that optimal unemployment insurance involves a decreasing sequence of insurance payment to the worker while he remains unemployed. Not surprisingly, this decreasing payment sequence is the efficient result in a moral hazard problem, which is caused by unobservable search effort. Next, we want to show that the unemployment benefit sequence decreases more mildly in our model than would occur in classical

models and it involves a discontinuity as we have in a real economy. In order to compare our model with classical models, I also provide a benchmark model in appendix [3], in which all settings are the same as our model except that consumption goods can be substituted freely.

PROPOSITION 4: Compared to the benchmark, the promised value in the next period is smaller before the commitment good is adjusted $\hat{v}' < \tilde{v}'$, if $u''' > 0$.

Proof. See appendix [5].

PROPOSITION 5: Before the commitment good is adjusted, the transfer from the government to the unemployed worker is higher ($\hat{\tau} > \tilde{\tau}$), compared to the benchmark.

Proof. From Proposition 3 we get $\hat{v}' < \tilde{v}'$. Lemma 1 in appendix [2] shows if $\hat{v}' < \tilde{v}'$, then $u(\hat{x}, y_{-1}) > u(\tilde{x}, \tilde{y})$. We want to prove that $\hat{\tau} > \tilde{\tau}$. Suppose not. By constraints $\hat{x} + y_{-1} = \hat{\tau}$ and $\tilde{x} + \tilde{y} = \tilde{\tau}$, since y_{-1} is a feasible choice when total transfer $\tilde{\tau} > \hat{\tau}$, but it is not chosen ($\tilde{y} \neq y_{-1}$). So we should have $u(\tilde{x}, \tilde{y}) > u(\hat{x}, y_{-1})$, which is a contradiction. Q.E.D.

Proposition 4 and 5 show that with consumption commitments government will transfer more but promise less to the unemployed worker. The intuition is that before the commitment good is adjusted, the transfer cannot be reduced much because the agent has to spend a fixed amount on commitment goods. If government lowers the payments too much, the unemployed worker would just cut back sharply on his consumption of adjustable goods. This would induce a large increase in marginal utility and create a large welfare cost, which couldn't be optimal. With commitment goods, government has to maintain a relatively higher level of unemployment benefit to the unemployed than would occur when all consumption goods can be adjusted flexibly. However, the higher payment is at the expense of future claim of consumption. So the best strategy with a commitment good is: because of the adjustment cost, the transfer should be higher in order to keep the commitment good, but claims for future consumption should be lower.

Proposition 6: With consumption commitment, search effort is higher compared to benchmark ($\hat{a} > \tilde{a}$).

Proof. From proposition 4 we have $\hat{v}' < \tilde{v}'$, then $\hat{a} > \tilde{a}$ follows immediately from (21). Q.E.D.

The logic of proposition 6 is as follows: Since the incentive for seeking a job comes from the difference between the utilities when employed and unemployed, a lower promised value in the next period means a heavier penalty for continuing to be unemployed, which will induce a greater search effort. From this property, we understand that with consumption commitments even if the agent gets more from government, he will not become lazier because of those higher payments. Instead, the agent has to exert a larger effort in seeking a new job.

In this section we have discussed some properties of the optimal contract. Now we proceed to solve a parameterized model numerically and see the evolution of the optimal unemployment insurance contract.

IV. Quantitative Analysis

Calibration

Now we give function forms and assign parameter values to the model and try to get some numerical results. The utility function takes the standard form

$u(x, y) = \frac{(x^\gamma y^{1-\gamma})^{1-\sigma}}{1-\sigma}$. We follow Hopenhayn (1997) to set the form of hazard function

$p(a) = 1 - e^{-ra}$, which is an exponential distribution with parameter r . We set $\sigma = 0.5$, giving an intermediate degree of risk aversion. This number may seem small relative to those used in the macro literature, which are generally above one. However, it should be taken into account that we are using weekly data for our calibration exercise, and the elasticity of substitution on weekly consumption is most probably larger than that corresponding to quarterly consumption.¹² γ is the share of total expenditure on adjustable goods while $(1 - \gamma)$ is consumption share of commitment goods. We take γ as 0.5. The lower gamma is, the larger proportion of commitment goods in the consumption bundle. We will do sensitivity analysis for different values of γ . The value of parameter

¹² Arguments from Hopenhayn (1997).

r is set by following the same procedure as Hopenhayn (1997) to match a 10% hazard rate as in Meyer (1990). The parameter r we get in this exercise is 4.8527×10^{-4} . Discount factor β is set to be 0.999, representing a yearly discount factor of 0.95. Adjustment cost K is set to 4 months' pay. Also, we will check sensitivity for adjustment cost in the next subsection. The wage rate for an employed worker is normalized to be 100.

Numerical Results

In this subsection, we use the calibration we get above to compute numerical results for two regimes: optimal contract with and without consumption commitments. The latter corresponds to the one in the classical model. We want to explore the evolution of the optimal unemployment insurance contract in the circumstance when consumption commitments are considered. The results are computed by setting the initial promised value to be the same as the one offered to the unemployed agent under the current unemployment policy at the beginning of the unemployment period.

1. Costs and Value of the Current Insurance Policy

The current insurance program is represented by a contract that provides 66% replacement ratio to the unemployed for 26 weeks and zero benefits afterwards. We can solve the cost and value of the current insurance program by using a backward induction method from week 26 to the beginning of the unemployment period. Let V_{26} denote the value function for an unemployed worker at week 26. Let V^{aut} represent the autarky value function for an unemployed worker with no transfers from the government. At week 26, the problem for the unemployed worker is to choose an effort level to maximize the expected utility function:

$$\begin{aligned}
 V_{26}^{UI} &= \max_{a_{26}^{UI}} \left[u(x_{26}^{UI}, y_{-1}) - a_{26}^{UI} + \beta(p(a_{26}^{UI})V^e + (1 - p(a_{26}^{UI}))V^{aut}) \right] \\
 s.t. & \\
 x_{26}^{UI} + y_{-1} &= T^{UI} \\
 V^{aut} &= \max_{a^{aut}} \left[u(0) - a^{aut} + \beta(p(a^{aut})V^e + (1 - p(a^{aut}))V^{aut}) \right]
 \end{aligned}$$

where T^{UI} is a fixed amount of transfers from the government.

When we solve this problem, we obtain the value function in period 26, which can be used for solving the value function and effort level in period 25. More generally, if we continue this recursion, we can solve for the cost and value function up to the first period of unemployment. The general form of the recursive problem is as follows:

$$V_t^{UI} = \max_{a_t^{UI}} [u(x_t^{UI}, y_{-1}) - a_t^{UI} + \beta(p(a_t^{UI})V^e + (1 - p(a_t^{UI}))V_{t+1}^{UI})]$$

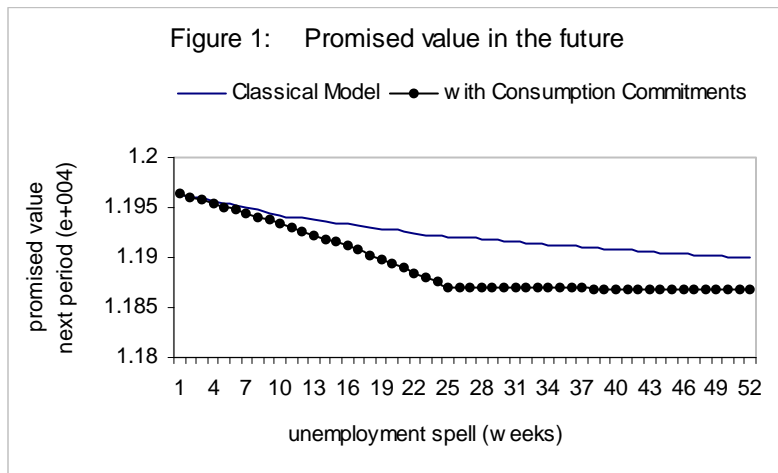
s.t.

$$x_t^{UI} + y_{-1} = T^{UI}$$

$$t = 25, 24, \dots, 1$$

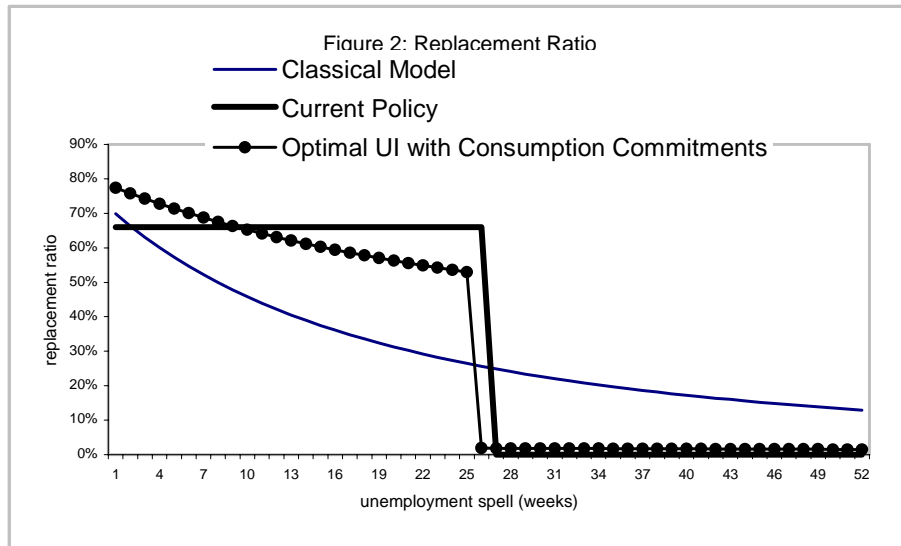
2. Optimal Contracts with and without Consumption Commitments

We compare the results of the two regimes and show the differences created by the introduction of commitment goods and adjustment costs. Figure 1 shows the sequence of promised values over the duration of unemployment in two models.¹³ The smooth line represents the promised utility values to the agent over unemployment spells. The decreasing trend is due to the incentive problem: considering unobservable job search effort, the optimal contract has to reduce the agent's claims for future consumption as punishment to induce appropriate effort level. With consumption commitments, the promised values specified in the optimal contract decrease faster than those in classical model before the commitment goods are adjusted. The kink corresponds to the moment when adjustment is made.



¹³ This graph is obtained with parameter $\gamma = 0.5$ and $K = 4.5$ months pay.

Figure 2 provides the main results of this paper. In this graph we plot the replacement ratio in three regimes: the classical model, the model with consumption commitment and the current policy. The smooth decreasing line illustrates the replacement ratio suggested by classical models. They argue that the optimal design of unemployment insurance involves a continuous decreasing replacement ratio throughout the unemployment spell. The bold line is the replacement ratio of the current unemployment insurance program, which is approximately a 66% replacement ratio for 6 months dropping to zero afterwards. From the graph we can see that there is a significant difference between the optimal UI contract provided by classical models and the current program. This is probably why the current unemployment insurance program has been widely criticized: first, the flat unemployment benefit cannot be efficient with unobservable search effort; second, the jump cannot be explained as an optimal allocation.



In this paper we show that the optimal contract involves a relatively flat decreasing replacement ratio for 25 weeks followed by a large drop. The dotted line represents the optimal replacement ratio we obtain. The intuition is as follows: with adjustment cost it is optimal for the commitment good not be adjusted at the beginning of unemployment periods. The unemployed are worse off than we have expected because they have to spend a fixed amount of the government transfer on commitment goods and consume adjustable goods with what remains. This will amplify preference risk aversion and demand for consumption smoothing, which requires the government to maintain a higher

level and flatter decreasing replacement ratio for a period of time. However, as unemployment duration increases, it will be optimal to pay adjustment costs and attempt to re-optimize the consumption bundle. The logic is that on the one hand the unemployment benefit should be reduced over time to induce incentives; on the other hand maintaining commitment goods with a decreasing transfer becomes more and more costly.¹⁴ So at some point it will be optimal to pay adjustment costs after comparing insurance benefits and costs.

When it's time to change commitment goods, a large drop in payment sequence occurs. Here the drop is due to two kinds of effects. One effect is the re-optimization of the consumption bundle: since people can freely balance between adjustable goods and commitment goods, the government doesn't have to give much to the individual. The other, which is more important, is reduced promised value. Figure 1 shows that the promised value decreases faster with commitment good than without. With consumption commitments, because of the sharp increase of marginal utility, transfer doesn't decrease much but promised value does. This means we don't have a match between transfer during this period and promised value in the future as we do in classical models. When it's time to change the commitment goods, the promised value has been reduced to a very low level. From proposition 2, in which we state the transfer is an increasing function of current promised value, we can expect the optimal transfer after the adjustment would be reduced to a very low level.

From the graph, we can see that the optimal contract in our model fits the current program well, which involves a relatively flat payment sequence and an abrupt discontinuity. Furthermore, we can also compute how close the optimal contract we obtained theoretically is to the current policy in practice. The expected costs of the optimal contract in theory and the contract under current policy can be obtained as follows:

¹⁴ Because of the concavity of utility function.

$$C(V_0^{UI}, y_{-1}) = \beta^{\bar{t}-1} K + \sum_{t=1}^{\bar{t}} \prod_{t=1}^{\bar{t}} (1 - p(\hat{a}_t)) \beta^{t-1} \hat{\tau} + \sum_{t=\bar{t}+1}^{\infty} \prod_{t=\bar{t}+1}^{\infty} (1 - p(a_t)) \beta^{t-1} \tau$$

$$C^{UI}(V_0^{UI}, y_{-1}) = \beta^{26} K + \sum_{t=1}^{26} \prod_{t=1}^{26} (1 - p(a_t^{UI})) \beta^{t-1} T^{UI}$$

where $C(V_0^{UI}, y_{-1})$ and $C^{UI}(V_0^{UI}, y_{-1})$ are the total costs of the optimal contract in our model and the current unemployment program respectively. \bar{t} is the time when commitment goods are adjusted in the optimal contract. \hat{a}_t, a_t and a_t^{UI} are optimal choices of search effort for the unemployed worker. V_0^{UI} is the promised welfare value that the current unemployment system provides at the beginning of a period of unemployment. Other notations are consistent with previous settings.

Table 1: Cost Savings

	Initial welfare level	Cost	Normalized Cost	Cost Saving
Current system	11967	1256.3	100%	0%
Optimal contract	11967	1234.8	98.29%	1.71%

Table 1 compares total costs of the optimal contract and the current system given the same initial welfare level. From these numbers, we can see that optimal contract is more efficient than current program in that it provides the same welfare but incurs less cost. The cost savings come from incentive efficiency that the current program cannot achieve with a sequence of flat unemployment benefits. This is also why the current program has been widely criticized. However, we find that with consumption commitment, the cost savings achieved by the optimal contract compared with current policy is very moderate, only 1.71%, which means our current unemployment program is very close to the optimal result we can achieve. Therefore our results give an explanation to justify current policy. It shows that current policy isn't as bad as researchers have traditionally believed. The difference between the optimal UI contracts we get theoretically and the one in practice is small and could be explained by administration cost.

Figure 3 compares effort level in the two models. Search effort is higher in the model with commitments than without. This is due to the sharp decreasing promised value, which induces a higher incentive to find a job. The result here is interesting: we don't lose efficiency in incentives even if a higher transfer to the worker is maintained. People

don't become lazier because of the transfers. On the contrary, they have to exert more effort in searching to keep the commitment good they previously made.

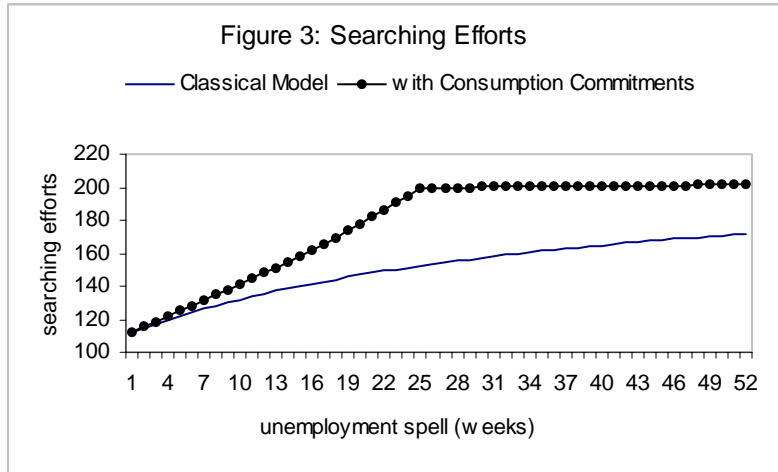
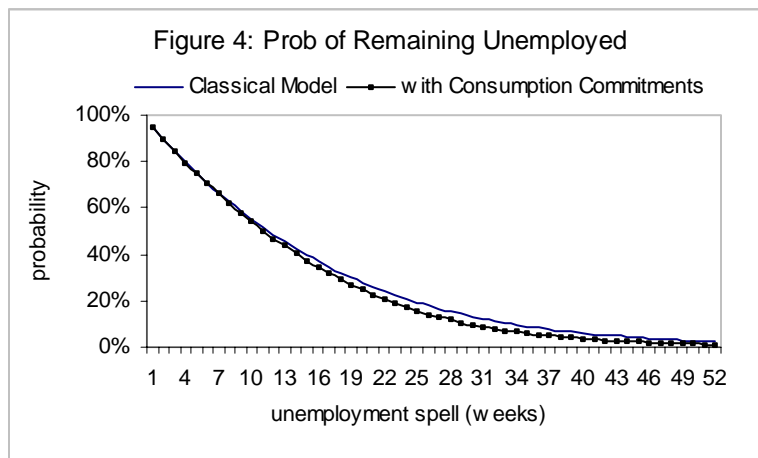


Figure 4 plots the probability of remaining unemployed for both models. The upper smooth line is the probability of remaining at unemployment status over time in the classical model. Since during every period the worker spends some amount of time searching for a job, the probability of remaining unemployed is decreasing over time. The lower dotted line represents that probability in the model with consumption commitment. The lower probability we get is consistent with the results in figure 3. The higher search effort will result in a lower probability of staying unemployed.

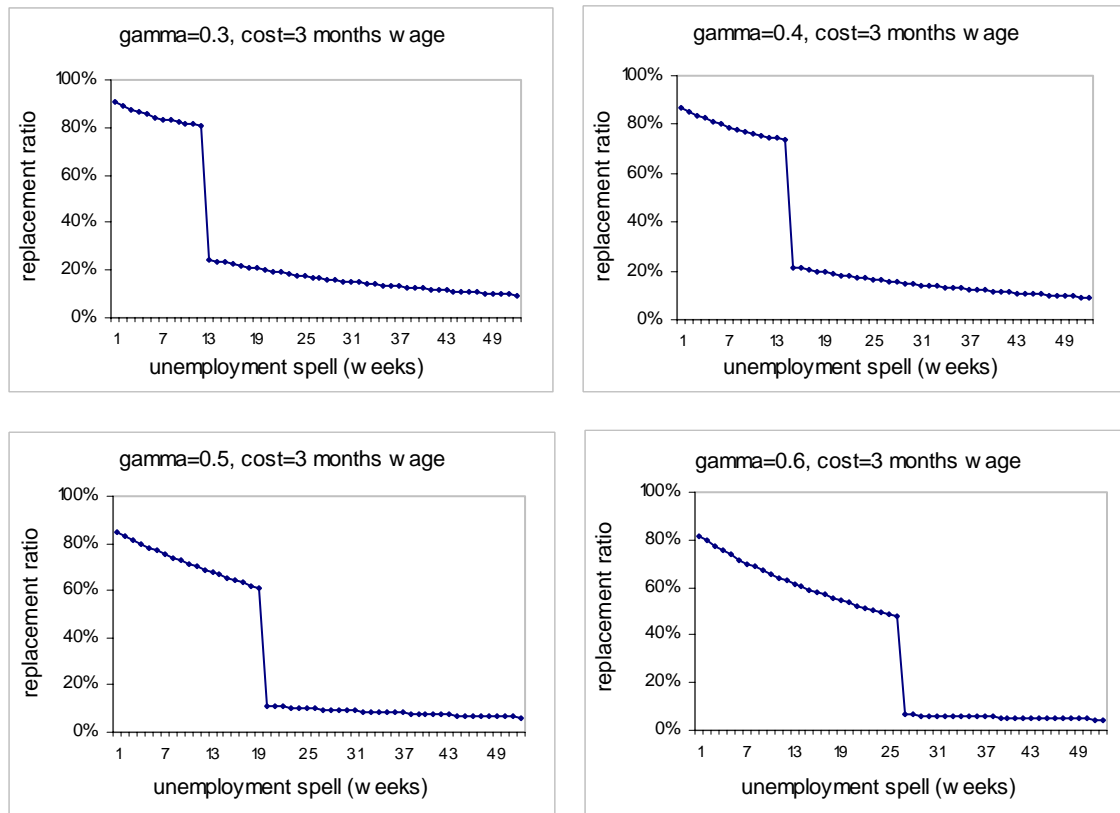


We also plot the graphs for consumption of both goods (commitment, adjustable) in the two models and put them in appendix [6].

Sensitivity Analysis

In this subsection we want to check if the properties of the optimal contract we get in the model are sensitive to changes in parameters. First, we check the sensitivity to change in consumption share γ . The higher γ means lower proportion of commitment goods in total expenditure. For different γ from 0.3 to 0.7, we plot optimal replacement ratio values in figure 5.

Figure 5

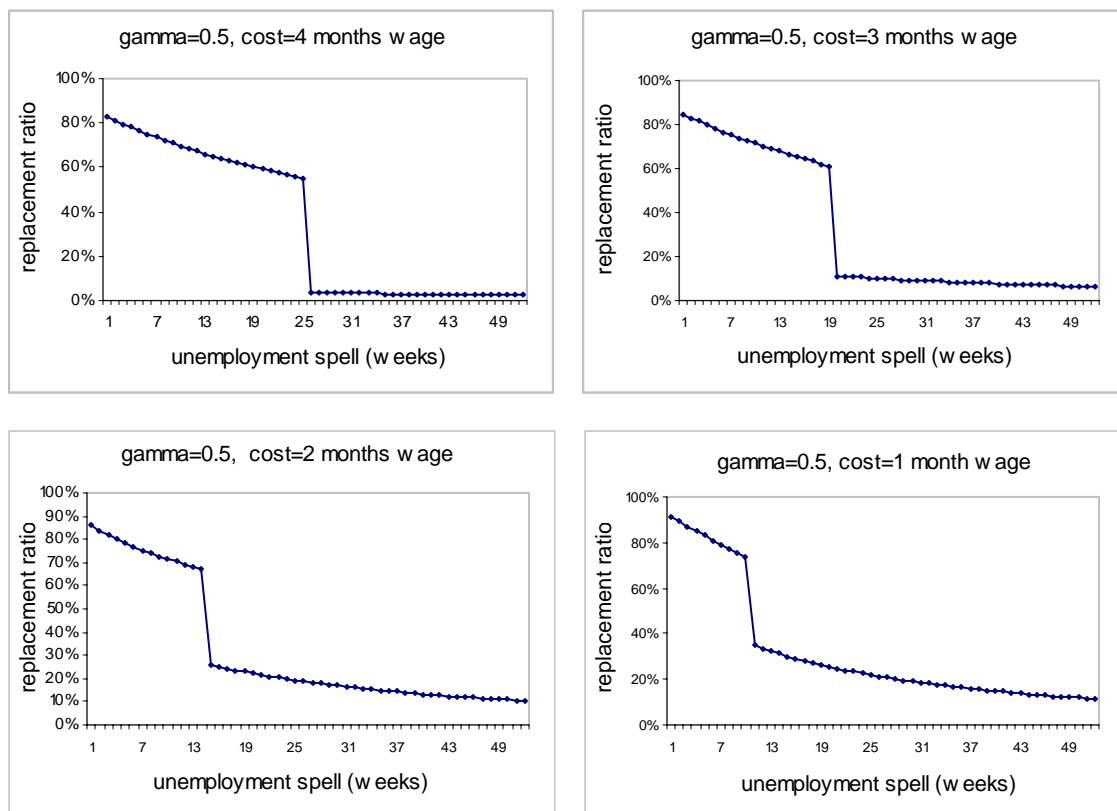


To see the effects due to change in consumption share, we fix the adjustment cost at 3 months' pay. From the graphs we can see that as γ increases two things need to be noticed. One is that the rate of decrease of the replacement ratio becomes steeper. The intuition is that when commitment goods account for a smaller proportion of total expenditure, the consumption bundle can be adjusted more flexibly; therefore unemployment benefits can be reduced more quickly. The other thing we can find is that the greater γ , the longer the period sustained before adjustment cost is paid. The logic here is that a smaller proportion of commitment goods mean people have more flexibility

to balance consumption, so the urge to adjust commitment goods will be smaller. From these graphs we get that, overall, the time and magnitude of the jump are not sensitive to change in γ .

Next we do sensitivity analysis on adjustment cost K . We choose the cost from 1 month's pay to 4 months' pay, while keeping the consumption share at 50% percent for both goods. We plot the results in figure 6. These graphs show that the smaller the cost, the sooner commitment goods are changed, which is very intuitive. Meanwhile we still see a large drop when adjustment occurs. Sensitivity analysis shows that our results are robust.

Figure 6



V. Conclusion and Extensions

Current unemployment insurance policy has been widely criticized because of the perverse effects it has on the incentives for reemployment and the discontinuity in its sequence of payments. Classical models argue that the optimal unemployment contract

should involve a continuous decreasing replacement ratio throughout the unemployment period. In this paper we incorporate consumption commitments into the optimal unemployment insurance design. Commitment goods amplify preference risk aversion so that demand for consumption smoothing becomes larger. The results we get are quite different from those of classical models. The optimal contract we obtained involves a relatively flat decreasing replacement ratio for 25 weeks followed by a large drop to a very low level of transfer. The results fit current policy well, which gives an explanation to justify the current unemployment insurance program in practice. Furthermore, we have another interesting finding: even with higher and more mildly decreasing unemployment benefits, the unemployed workers don't become lazier. On the contrary, their incentives to find a job are higher if they want to keep the commitment goods they previously had. The implication here is that we don't lose efficiency in incentives even if a higher transfer to the worker is maintained. In this paper, we show that current policy isn't as bad as researchers have traditionally believed. The cost savings if we change from current policy to the optimal contract are only 1.7%, which means that the current unemployment program is very close to the optimal result we can achieve. The difference between the optimal contracts we get in theory and the current policy in the real world is small and could be explained by administrative costs.

The first extension of this paper is to add assets into the model and allows for saving behavior. In our model, we solve the social planner's problem in which the adjustment cost is taken as a social cost. However in practice it is the individual who makes the decision to change consumption. And adjustment cost usually takes the form of asset devaluation. For example, the transaction cost of selling a house is a kind of decrease in the asset value. Another example is selling a car; the cost is the difference between the market resale value and the actual value. Since so far my model doesn't account for assets, such devaluation cost cannot be reflected in the budget constraint. So what we do is to add assets into the model and decentralize the problem. The second extension is related to the first one: when government doesn't have the ability to control the agent's consumption behavior, we have to consider the incentive compatibility to induce the individual to make adjustment at the right time. Another extension deals with the discounted utility value for an employed worker. In our model we assume the wage rate

doesn't depend on unemployment history but most literature considers the wage rate to be a function of the worker's previous unemployment experience. We are going to relax this assumption in the future. In addition, we are going to do some welfare analysis to compare the classical model, our model and the current policy.

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APPENDIX

[1] After the commitment good is adjusted, we can get the first-order conditions:

$$L = \tau + \beta(1 - p(a))W(v') + \lambda \left\{ v - \left[u(\tau - y, y) - a + \beta(p(a)v^e + (1 - p(a))v') \right] \right\} \\ + \eta\beta \left[1 - p'(a)(v^e - v') \right]$$

F.O.C.

$$W'(v') = \lambda + \eta \frac{p'(a)}{1 - p(a)} = \frac{1}{u_1(\tau - y, y)} - \eta \frac{p'(a)}{1 - p(a)} \quad (\text{A1})$$

$$\beta p'(a)W(v') - \eta\beta p''(a)(v^e - v') = 0 \quad (\text{A2})$$

$$u_1(\tau - y, y) = u_2(\tau - y, y) \quad (\text{A3})$$

Envelope condition:

$$W'(v) = \lambda = \frac{1}{u_1(\tau - y, y)} = W'(v') + \eta \frac{p'(a)}{1 - p(a)} \quad (\text{A4})$$

[2] Proof of Lemma 1.

Proof. If $\hat{v}'_1 > \hat{v}'_2$ the $u(\hat{x}_1, y_{-1}) < u(\hat{x}_2, y_{-1})$; If $v'_1 > v'_2$ the $u(x_1, y_1) < u(x_2, y_2)$, given hazard rate function $p(a) = 1 - e^{-ra}$.

Proof. From (9) $u(\hat{x}, y_{-1}) - \hat{a} + \beta(p(\hat{a})v^e + (1 - p(\hat{a}))\hat{v}') = v$

$$\Rightarrow u(\hat{x}, y_{-1}) - \hat{a} + \beta(v^e - (1 - p(\hat{a}))(v^e - \hat{v}')) = v$$

$$\Rightarrow u(\hat{x}, y_{-1}) - \hat{a} + \beta(v^e - \frac{p'(\hat{a})}{r}(v^e - \hat{v}')) = v \quad (\text{A5})$$

$$\Rightarrow u(\hat{x}, y_{-1}) - \hat{a} + \beta(v^e - \frac{1}{\beta r}) = v \quad (\text{A6})$$

where (A5) is got given $p(a) = 1 - e^{-ra}$, while (A6) uses $\beta p'(\hat{a})(v^e - \hat{v}') = 1$ (21).

If $\hat{v}'_1 > \hat{v}'_2$, from (21) we have $\hat{a}_1 < \hat{a}_2$. From (A6), we get $u(\hat{x}_1, y_{-1}) < u(\hat{x}_2, y_{-1})$. The second part of lemma 1 can be proved similarly. Q.E.D.

[3] Proof of Proposition 3

Proof. We want to prove if $v_1 < v_2$, then $\hat{v}'_1 < \hat{v}'_2$. Suppose not. From Lemma 1, we get

$$u(\hat{x}_1, y_{-1}) < u(\hat{x}_2, y_{-1}), \text{ therefore } \hat{x}_1 < \hat{x}_2 \Rightarrow \frac{1}{u_1(\hat{x}_1, y_{-1})} < \frac{1}{u_1(\hat{x}_2, y_{-1})} \quad (\text{A7})$$

From (23) we get $\hat{\eta} = \frac{p'(\hat{a})C(\hat{v}', y_{-1})}{-p''(\hat{a})(v^e - \hat{v}')}$

From (21) $\beta p'(\hat{a})(v^e - \hat{v}') = 1 \Rightarrow \hat{\eta} = \frac{[p'(\hat{a})]^2 C(\hat{v}', y_{-1})}{-p''(\hat{a})}$

From (22), we have $C'(\hat{v}', y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \frac{\beta \eta p'(\hat{a})}{(1 - p(\hat{a}))}$

$$\begin{aligned} \Rightarrow C'(\hat{v}', y_{-1}) &= \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \beta \frac{[p'(\hat{a})]^3 C(\hat{v}', y_{-1})}{-p''(\hat{a})(1 - p(\hat{a}))} \\ &= \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \beta p'(\hat{a}) C(\hat{v}', y_{-1}) \quad (\text{given } p(a) = 1 - e^{-ra}) \\ \Rightarrow \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} &= C'(\hat{v}', y_{-1}) + \beta p'(\hat{a}) C(\hat{v}', y_{-1}) \quad (\text{A8}) \end{aligned}$$

From (A10) we get

$$C'(\hat{v}'_1, y_{-1}) + \beta p'(\hat{a}_1) C(\hat{v}'_1, y_{-1}) < C'(\hat{v}'_2, y_{-1}) + \beta p'(\hat{a}_2) C(\hat{v}'_2, y_{-1}) \quad (\text{A9})$$

Since we suppose $\hat{v}'_1 > \hat{v}'_2$, from (21) $\beta p'(\hat{a})(v^e - \hat{v}') = 1$, we can get $p'(\hat{a}_1) > p'(\hat{a}_2)$. Because the cost function is increasing and convex, we should have $C'(\hat{v}'_1, y_{-1}) + \beta p'(\hat{a}_1) C(\hat{v}'_1, y_{-1}) > C'(\hat{v}'_2, y_{-1}) + \beta p'(\hat{a}_2) C(\hat{v}'_2, y_{-1})$, which contradicts with (A9). Q.E.D.

[4] Benchmark Model

$$C(v) = \underset{\tilde{x}, \tilde{y}, \tilde{\tau}, \tilde{a}, \tilde{v}'}{\text{Min}} \{ \tilde{\tau} + \beta(1 - p(\tilde{a}))C(\tilde{v}') \}$$

(A5)

$$s.t. \quad u(\tilde{x}, \tilde{y}) - \tilde{a} + \beta(p(\tilde{a})v^e + (1 - p(\tilde{a}))\tilde{v}') \geq v \quad (\text{A10})$$

$$\tilde{a} \in \arg \max \{ u(\tilde{x}, \tilde{y}) - \tilde{a} + \beta(p(\tilde{a})v^e + (1 - p(\tilde{a}))\tilde{v}') \} \quad (\text{A11})$$

$$\tilde{x} + \tilde{y} = \tilde{\tau} \quad (\text{A12})$$

where v^e is defined as:

$$v^e = \frac{u(x^*, y^*)}{1 - \beta}$$

$$s.t. \quad u_1(x^*, y^*) = u_2(x^*, y^*) \text{ and } x^* + y^* = w$$

[5] Proof of Proposition 4.

Proof. As in proof of Proposition 3, we get

$$C'(\hat{v}', y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \beta p'(\hat{a}) C(\hat{v}', y_{-1}) \quad (\text{given } p(a) = 1 - e^{-ra})$$

$$\Rightarrow \beta p'(\hat{a})C(\hat{v}', y_{-1}) = C'(v, y_{-1}) - C'(\hat{v}', y_{-1}) = \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \frac{1}{u_1(\tilde{\tau}' - y_{-1}, y_{-1})} \quad (\text{A13})$$

Similarly, we can get

$$\beta p'(\tilde{a})C(\tilde{v}') = C'(v) - C'(\tilde{v}') = \frac{1}{u_1(\tilde{\tau} - \tilde{y}, \tilde{y})} - \frac{1}{u_1(\tilde{\tau}' - \tilde{y}', \tilde{y}')} \quad (\text{A14})$$

Suppose $\hat{v}' = \tilde{v}'$. Then from Lemma 1, we have $u(\hat{\tau} - y_{-1}, y_{-1}) = u(\tilde{\tau} - \tilde{y}, \tilde{y})$. Since the given level of y_{-1} satisfies $y_{-1} > \tilde{y}$, we have $(\hat{\tau} - y_{-1}) < (\tilde{\tau} - \tilde{y})$. Since we suppose the best solution to a given state (v, y_{-1}) is that $\tilde{v}' = \hat{v}'$, we get $\tilde{v}'' = \hat{v}''$. Again from Lemma 1, we have $u(\hat{\tau}' - y_{-1}, y_{-1}) = u(\tilde{\tau}' - \tilde{y}', \tilde{y}')$. Similarly, we get

$(\hat{\tau}' - y_{-1}) < (\tilde{\tau}' - \tilde{y}')$. Because $\tilde{y}' < \tilde{y}$ and utility function is strictly concave, then we get $[(\tilde{\tau} - \tilde{y}) - (\hat{\tau} - y_{-1})] < [(\tilde{\tau}' - \tilde{y}') - (\hat{\tau}' - y_{-1})]$.

$$\begin{aligned} & \beta p'(\tilde{a})C(\tilde{v}') - \beta p'(\hat{a})C(\hat{v}', y_{-1}) \\ &= \left\{ \frac{1}{u_1(\tilde{\tau} - \tilde{y}, \tilde{y})} - \frac{1}{u_1(\tilde{\tau}' - \tilde{y}', \tilde{y}')} \right\} - \left\{ \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} - \frac{1}{u_1(\hat{\tau}' - y_{-1}, y_{-1})} \right\} \\ &= \left\{ \frac{1}{u_1(\tilde{\tau} - \tilde{y}, \tilde{y})} - \frac{1}{u_1(\hat{\tau}' - y_{-1}, y_{-1})} \right\} - \left\{ \frac{1}{u_1(\tilde{\tau}' - \tilde{y}', \tilde{y}')} - \frac{1}{u_1(\hat{\tau} - y_{-1}, y_{-1})} \right\} \end{aligned} \quad (\text{A15})$$

If we assume $u''' > 0$, then (A15) < 0, which implies $C(\tilde{v}') - C(\hat{v}', y_{-1}) < 0$, where $\tilde{v}' = \hat{v}'$.

Contradiction. Since $\beta p'(\hat{a})C(\hat{v}', y_{-1}) = C'(v, y_{-1}) - C'(\hat{v}', y_{-1})$ is too small when $\tilde{v}' = \hat{v}'$, we should have $\tilde{v}' > \hat{v}'$, given the convexity of cost function. Q.E.D.

[6]

